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Algoritmos para estimar la frecuencia instantánea de una secuencia respiratoria variable en el tiempo

Algorithms to estimate the instantaneous frequency of a respiratory time-varying sequence

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Abstract: ***In various occasions, algorithms to estimate instantaneous frequency from a cyclic (seasonal) sequence to detect slow changes are needed. That is the case of the estimation of the variations of the respiratory rate for diagnostic purposes. There are a few possible procedures to estimate such an instantaneous frequency, but they have not been thoroughly assessed in order to select the best for a respiration rate estimation from a volumetric surrogate signal. This paper discusses the implementation of some algorithms for instantaneous frequency estimation in MATLAB and compares their performance from known synthetic signals resembling real-world respiratory signals, by using goodness of fit parameters. In detail, a method based on the first conditional spectral moment of the time-frequency distribution of the input signal x, another using the derivative of the phase of the analytic signal of x (found using the Hilbert transform), and others based on second-order auto-regressive models. Goodness of fit (maximum absolute error and mean-squared error) between the estimated and the expected ideal instantaneous frequencies were computed. The root MUSIC algorithm outperforms the others under assessment, showing its superiority for instantaneous respiratory frequency estimation from a volumetric surrogate signal.***

Keywords:* *Respiratory signals, Instantaneous frequency estimation, Time-frequency distributions, Hilbert transform, Auto-regressive models, root MUSIC

1. Introduction

In various occasions, algorithms to estimate instantaneous frequency from a cyclic (seasonal) sequence to detect slow changes are needed. Examples of this kind of situations comprise the estimation of the variations of the respiratory rate or the pitch variation during a sustained vowel phonation for diagnostic purposes; or the estimation of the angular velocity of an engine for speed control or fault detection; as well as the estimation of seasonal component variations in time series of financial/economic or environmental/climatic events.

1.1. Respiratory movement signals and respiration rate

The time-varying respiration rate (RR) or instantaneous respiratory frequency is an important variable to characterize the physiological condition of a subject in clinical and fitness applications. To acquire the respiration primary information, from which to estimate the instantaneous frequency, numerous measurement methods have been used. For instance, spirometry directly measures the flow rate of the breathing air, but estimating certain parameters of the breathing airflow (e.g. temperature, pressure, humidity and CO_2 concentration) can also provide alternative procedures [2]. Nasal or oral/nasal thermistors [21], or infrared thermography [1] can measure the temperature of the inhaling (cooler) and exhaling (warmer) air, as well as mouthpieces or facemasks can be used for monitoring the associated pressures [19]. Electrical impedance pneumography [6], [9], [23], inductance pneumography [7], and capnography [22] have been extensively used, along with signals as ECG [11], [13], and PPG [12], [14] from which respiration movement can be also derived. More recently, volumetric surrogate signals have been acquired using radars [15], [17], or videocams [18], [20]. High correlation indexes are expected between the respiratory movement signals obtained by the different monitoring methods.

This respiratory movement signal oscillates at around 12 breath cycles per minute (0.2 Hz), continuously varying in accordance with several conditions. The upper plot of Figure 1 shows that this signal, here obtained with a temperature sensor, is normally affected by slow-drifts or trends, and distortions in the waveform, but it can be still considered as a mono-component and the instantaneous frequency successfully shows the evolution of the frequency content of this respiratory movement signal with time. From that signal, the instantaneous respiratory frequency or respiration rate (lower plot in Figure 1) can be then estimated by a proper method, showing the variations in time.

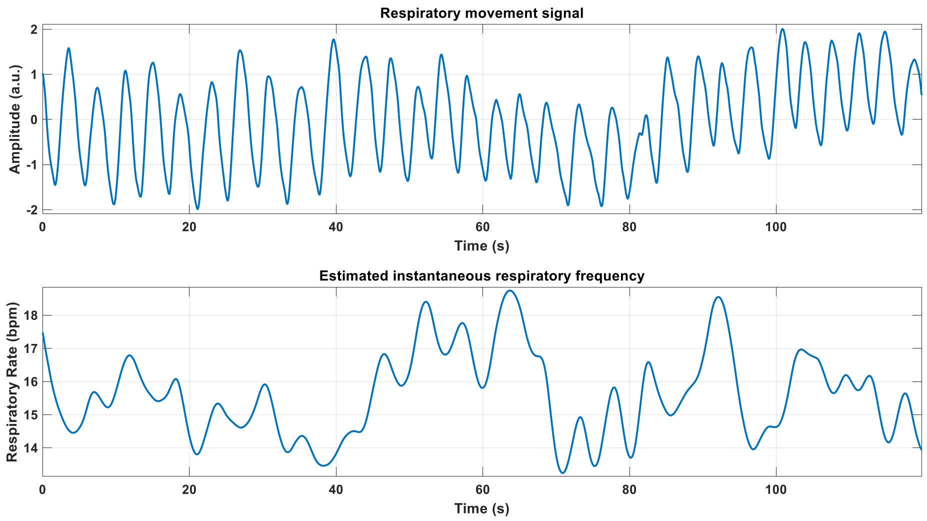

Figure 1. Respiratory movement signal (sampled at 25 sps) obtained with a temperature sensor (apparently, a thermistor) around the mouth and the nose of a subject (plot on top) and the instantaneous respiratory frequency (respiration rate) estimated from it (plot on the bottom) [20].[1]

Although the instantaneous frequency estimation problem has been studied in general [4], [5], [16], or some other specific contexts even nowadays [10], it has not been sufficiently addressed in regard to respiration rate estimation with the exception of from electrical impedance pneumography signals [9]. This work tries to select 'the best' approach for estimating the instantaneous respiratory frequency from a volumetric surrogate signal.

[1] Created by the authors, by using signals provided for the CLAIB 2019 Scientific Challenge, Cancun, Mexico, Oct. 3-5, 2019.

2. Methodology

There are several approaches that have been used to estimate the instantaneous frequency in different environments [4], [16], [10], including respiratory rate [9], [11], [12], [13], [14], [18], [23]. Most of those algorithms can be easily implemented and used in MATLAB, including: an algorithm based on the analytic signal, and one which estimates the instantaneous frequency as the first conditional spectral moment of the time-frequency distribution of the input signal, both embedded in the instfreq.m function of MATLAB. For this work, some other functions were implemented in MATLAB, comprising: one based on peaks detection in the time domain, several ones based on auto-regressive (AR) models and on the Eigenvectors and the root MUltiple SIgnal Classification (MUSIC) algorithm.

To compare the implemented algorithms for instantaneous respiratory frequency estimation, some synthetic signals where generated resembling the real-world respiratory volume surrogated signals, from where to estimate the known respiration rates. The measures for assessment were the maximum absolute error and the mean-squared error.

2.1. Algorithm based on the analytic signal (Hilbert)

Despite certain criticism [4], [8], the analytic signal via the Hilbert transform is still used to compute the instantaneous frequency. For any real-valued signal, $x(t)$, an analytical signal, $x_a(t)$, is associated as a complex-valued signal defined as

$$x_a(t) = x(t) + j\mathrm{HT}(x(t)), \tag{1}$$

where $\mathrm{HT}(x)$ is the Hilbert transform of x. In the frequency domain, X_a , which is the Fourier transform of x_a , is a single-sided Fourier transform, i.e. with the negative frequency values removed, doubling the strictly positive ones, and the DC component unchanged:

$$X_a(\nu) = \begin{cases} 0, & \text{if } \nu < 0 \\ X(0), & \text{if } \nu = 0 \\ 2X(\nu), & \text{if } \nu > 0 \end{cases}. \tag{2}$$

Therefore, the analytic signal, x_a , can be obtained from the real signal, $x(t)$, by forcing to zero its spectrum for the negative frequencies, without altering the information content since, for a real signal, $X(-\nu) = X^*(\nu)$.

It is then possible to define in a unique way the concept of instantaneous frequency, $f_{inst}(t)$, by the first-derivative of the argument or phase of the analytic signal as:

$$f_{inst}(t) = \frac{1}{2\pi}\frac{d\arg(x_a(t))}{dt}. \quad (3)$$

This method is inside the MATLAB function instfreq.m, as 'hilbert' 'Method', as well as in a function of similar name in the Time-Frequency Toolbox [3].

2.2. Algorithm based on the moment of the time-frequency distribution

The 'tfmoment' 'Method' in the instfreq.m MATLAB function estimates the instantaneous frequency as the first conditional spectral moment of the time-frequency distribution of the input signal. This method computes the spectrogram power spectrum, $P(t,f)$, of the input using the pspectrum.m function, which uses the spectrum as a time-frequency distribution and estimates the instantaneous frequency as

$$f_{inst}(t) = \frac{\int_0^\infty fP(t,f)df}{\int_0^\infty P(t,f)df}. \quad (4)$$

Remember that the spectrogram is defined as the squared modulus of the Short-Time Fourier Transform (*STFT*) and can be interpreted as a measure of the energy signal contained in the *t-f* domain, centred on (t,f). The Short-Time Fourier Transform is:

$$STFT_x(t,f;w) = \int_{-\infty}^{\infty} x(\tau)w^*(\tau - t)e^{-j2\pi ft}d\tau, \quad (5)$$

where $x(t)$ is the input signal in the time domain, $X(f)$ is its equivalent representation in the frequency domain, and $w(t)$ is the short-time analysis window around t=0 and f=0 ($W(f)$ is its version in the frequency domain). The *STFT* can be thought of as a local spectrum of the signal $x(\tau)$ around t.

For a sampled signal, $x[n]$, as in this case of the respiratory movement sequence, the *STFT* is efficiently implemented by using the FFT algorithm as

$$STFT_x[n,m;w] = \sum_k x[k]\, w^*[k-n]e^{-j2\pi mk},\ -\frac{1}{2} \le m \le \frac{1}{2}, \quad (6)$$

where k is the step with which w is displaced to window the data and it must be a multiple of the sampling period.

2.3. Algorithms based on peaks detection in the time domain

An intuitive way of estimating the instantaneous frequency consists of counting the number of peaks of the respiration movement signal in a known time interval and computing how many may occur in a 1-min window, assuming that the frequency is constant in that period of time. Actually, the time interval between two consecutive peaks (e.g. t_1 between the 1st and the 2nd peaks in Figure 2) is measured and then, the frequency is computed as its inverse (multiplied by 60 to express it in breaths per minute, bpm):

$$f_{inst}(t_1) = \frac{60}{t_1}\text{, in bpm.} \tag{7}$$

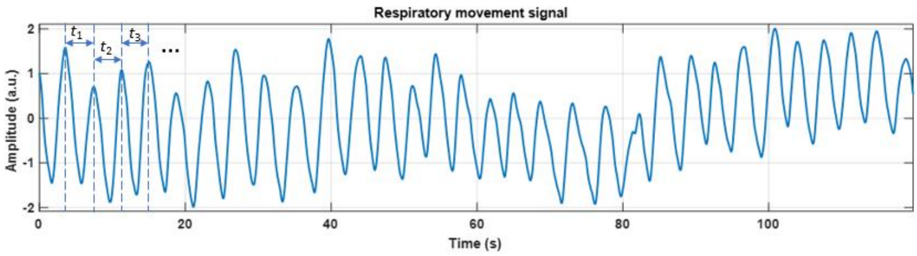

Figure 2. Intervals between consecutive peaks in the respiratory movement signal.[2]

The original estimated instantaneous frequencies coincide with the positions right between the peaks. To assign values of instantaneous frequency to other positions, some kind of interpolation is needed. We solved this by using the interp1.m MATLAB function with several interpolation methods:

- Linear interpolation (‘linear’), where the interpolated value at a query point is based on linear interpolation of the values at neighboring grid points in each respective dimension.
- Nearest neighbor interpolation (‘nearest’), where the interpolated value at a query point is the value at the nearest sample grid point.

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- Next neighbor interpolation ('next'), where the interpolated value at a query point is the value at the next sample grid point.
- Previous neighbor interpolation ('previous'), where the interpolated value at a query point is the value at the previous sample grid point.
- Modified Akima cubic Hermite interpolation ('makima'), where the interpolated value at a query point is based on a piecewise function of polynomials with degree at most three.
- Spline interpolation using not-a-knot end conditions ('spline'), where the interpolated value at a query point is based on a cubic interpolation of the values at neighboring grid points in each respective dimension.
- Shape-preserving piecewise cubic interpolation ('pchip'), where the interpolated value at a query point is based on a shape-preserving piecewise cubic interpolation of the values at neighboring grid points. This is used as default here.

2.4. Algorithms based on Auto-Regressive (AR) models

The respiratory movement sequence exhibits serial autocorrelation; i.e. linear association between lagged observations. The autoregressive (AR), or process models the conditional mean of $x(t)$ as a function of past observations, $x(t-1)$, $x(t-2)$,…, $x(t-p)$. An AR model of degree p, AR(p), depends on p past observations. The AR equation may model spectra with narrow peaks by placing zeroes of the A polynomial ($A(w)=1+a_1e^{-jw}+\ldots+a_pe^{-jpw}$) in

$$\phi(w) = \left|\frac{1}{A(w)}\right|^2 \sigma^2, \tag{8}$$

close to the unit circle.

The respiration movement signal could be considered as a mono-component signal and, therefore, it can be modeled with an AR(2). To estimate the parameters of the AR signal models, a system of linear equations is solved, guarantying the stability of the estimated AR polynomial. In this work, we implemented in MATLAB:

- The Burg's lattice-based method ('burg'), which solves the lattice filter equations using the harmonic mean of forward and backward squared prediction errors.

- The geometric lattice approach ('gl'), which is similar to Burg's method, but uses the geometric mean instead of the harmonic mean during minimization.
- The least-squares approach ('ls'), which minimizes the standard sum of squared forward-prediction errors.
- The Yule-Walker approach ('yw'), which solves the Yule-Walker equations, formed from sample covariances.
- The forward-backward approach ('fb'), which minimizes the sum of a least-squares criterion for a forward model, and the analogous criterion for a time-reversed model. This algorithm is used by default.

The information about the data outside the measured time interval (past and future values), is specified as:

- Post-windowing ('pow'), where missing end values are replaced with zeros and the summation is extended to time $N+n$ (N is the number of observations).
- Pre-windowing ('prw'), where missing past values are replaced with zeros and, therefore, the summation in the criteria can start at time equal to zero.
- Pre- and post-windowing ('ppw'), which is used in the Yule-Walker approach.
- No windowing ('now'), where only measured data is used to form regression vectors. The summation in the criteria starts at the sample index equal to $n+1$. This is the default value, except for 'yw'.

For this AR(2) model,

$$f_{inst}(t) = \frac{f_s}{2\pi} \cos^{-1}\left(\frac{a_1}{2\sqrt{-a_2}}\right). \quad (9)$$

With this method, we compute the instantaneous frequency for an interval of T samples every dt samples. The length of the analysis window (T) is not critical here, but we suggest a number of samples close to the expected period of the respiration cycle, i.e. T = 128, which is around 5 s at f_s = 25 sps (one period of the common 12-bpm breathing). The updating interval should be around 1 s, assuring an overlapping of about 80 % of two consecutive analysis windows. After this step, an interpolation method similar to those in the algorithm based on peaks (section 2.3) is used.

First of all, an optional decimation process, which includes a low-pass filtering (FIR window method with a Hamming window of length 30), could reduce the number of samples in the analysis window, in order to better catch the dynamics of the signal with the AR(2) model.

2.5. Approach based on the root MUSIC algorithm

The root MUSIC algorithm, originally proposed as an improvement to Pisarenko's method, can estimate the instantaneous frequency of a mono-component signal as the respiration movement using an eigenspace, which assumes that the signal comprises p = 2 complex exponentials (one for positive frequency and one for negative frequency) in the presence of Gaussian white noise. The eigenvalues are sorted in decreasing order, thus the eigenvectors corresponding to the 2 largest eigenvalues span the signal subspace and the rest is considered only noise. The frequency estimation function here is:

$$\hat{P}\left(e^{j\omega}\right) = \frac{1}{\sum_{i=p+1}^{M} \left|e^{H} v_i\right|^2}, \tag{10}$$

where p is the number of complex exponentials (2 here), M is the order of the autocorrelation matrix of the signal ($M \times M$ autocorrelation matrix of the respiration movement signal, R_{rm}), the v_i are the eigenvectors of noise and e is the steering vector,

$$e = \left[1, e^{j\omega}, e^{j2\omega}, \ldots\ e^{j(M-1)\omega}\ \right]^T. \tag{11}$$

By locating the p largest peaks of the estimation function $\hat{P}$, sometimes called the pseudospectrum, the frequency estimates for the p components are found. In this case, the positive frequency is kept and the negative one is ignored.

Pisarenko's method uses only a single eigenvector and takes a set of autoregressive coefficients to find its zeros analytically or with polynomial root finding algorithms. Root MUSIC, however, assumes that zeros may not be present and locates local minima instead, by searching the estimation function for peaks, which can be evaluated for any frequency, not just those of DFT bins, estimating instantaneous frequencies with accuracy higher than one sample, i.e. it is a super-resolution method.

For this implementation, similar to that of the AR models, we used an analysis window of *T* length (around 5 s), an updating period of *dt* length (around 1 s) and an interpolation stage to get an instantaneous respiratory frequency signal at the same sampling frequency as the respiration movement signal (f_s = 25 sps in this experiments). We could use a previous decimation stage as well.

2.6. Evaluation of the algorithms

For the evaluation of the implemented algorithms for instantaneous respiration frequency estimation, we used sets of 1000 runs with synthetic signals from the simplest to the most complete realistic model. First, we generated segments of 2 min, 0.2-Hz sinusoids (12 bpm), sampled at 25 sps, and with certain low-level noise (with high signal-to-noise ratio, around 10:1, though) and with a random original phase.

Then, we ran experiments again, all the implemented algorithms competing to estimate the instantaneous frequency in more realistic scenarios, including:

- different signal-to-noise ratios (SNR around 100:1 and 10:1),
- baseline wandering (15 times slower than the main frequency of 0.2 Hz),
- along with several levels of (amplitude and frequency) modulations at known variation laws:
    - sinusoids about 0.02 Hz, producing variations of ± 6 bpm by frequency modulation,
    - and amplitude changes of around 30 %.

All the estimated instantaneous frequency signals (obtained by the different algorithms) were compared with the known generating law of the instantaneous frequency by using the maximum absolute error (*MAE*) and the mean-squared error (*MSE*) according to:

$$MAE = \max(|x(t) - \hat{x}(t)|)\,, \qquad (12)$$

and

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\big(x(t) - \hat{x}(t)\big)^2, \qquad (13)$$

where $x(t)$ is the ideal (known) instantaneous frequency signal and $\hat{x}(t)$ is the estimated instantaneous frequency signal by the algorithm under assessment, and N is the number of samples in the 2-min segment (around 3 000 samples at 25 Hz).

At the end, we used the best algorithms to find the instantaneous frequency signals from real-world respiratory movement signals acquired from two subjects (one male and one female). Subjects were performing 3 different respiratory maneuvers: with controlled respiration at a fixed rate of 12 breaths per minute (i.e. 0.2 Hz, following a metronome), still and moving back and forth, and with spontaneous frequency respiration as well.

3. Results and discussion

In this section, we discuss the results of the instantaneous frequency estimators under assessment in the presence of ideal sinusoidal signals, in the presence of more realistic signals resembling the respiratory volume surrogated signals, and in the presence of real-life signals.

3.1. Instantaneous frequency estimators in the presence of ideal sinusoidal signals

For the ideal clean sinusoid of constant instantaneous frequency (f_o = 0.2019 Hz, i.e. 12.114 bpm), sampled at much higher sampling frequency (f_s = 25 Hz), there are several approaches that reach a perfect estimation of the instantaneous frequency during the whole 2-min sequence. In this experiment, with 1 000 repetitions of random initial phase, the forward-backward auto-regressive (AR) approach and the least-square AR, with no windowing, as well as the root MUSIC algorithm, got zero mean-squared error and zero maximum absolute error.

Most of the algorithms under assessment acceptably estimate the instantaneous frequency. However, even for this easy signal, some well-established methods had some troubles. The method based on the first conditional spectral moment of the time-frequency distribution (TF moment) and the one based on the derivative of the phase of the analytic signal using Hilbert transform (Hilbert) show a kind of oscillatory behavior around the perfect estimation value during the 2-min sequence. The instantaneous frequency estimated by the spectrogram, which is based on the Short-Time Fourier

Transform, depends on the specific segment of the signal analyzed , that is, it is not time-invariant. Similar behavior, but even more pronounced, exhibits the method that computes the derivative of the phase of the analytic signal. The maximum absolute error of the TF moment method is above 1.9 bpm, with a mean squared error in the order of 0.9 bpm^2, while the maximum absolute error of the Hilbert method is 0.6 bpm, with a mean squared error of 0.01 bpm^2. Recall that for this simple signal, the forward-backward AR approach with no windowing, the least-square AR with no windowing, and the root MUSIC algorithm perfectly performed.

3.2. Instantaneous frequency estimators in the presence of more realistic signals

The AR methods are seriously affected by more realistic respiratory sequences, that could contain combinations of additive slow-drifts or trends, certain amplitude modulation and frequency modulation, as well as some additive noise. The 'high-frequency' additive noise and the 'low-frequency' drift can be attenuated before the instantaneous frequency estimation, by using proper bandpass filtering in the band of approximately 0.1 to 0.6 Hz, where the respiration rate appears (i.e. between 6 and 36 bpm), and/or using detrending techniques as the empirical mode decomposition [18], [23]. However, certain levels of them will remain, affecting the estimation. The amplitude modulation can be seen as a multiplicative effect and it is almost impossible to get rid of it, and the frequency modulation carries the useful information of the time-varying instantaneous frequency.

Different experiments, with 1 000 runs each, were performed at different SNR and different levels of (amplitude and frequency) modulations, in which, consistently, three algorithms under test outperform the rest. These algorithms were Hilbert, TF moment and root MUSIC ranked in that very order, being the root MUSIC the best of all.

Actually, for these realistic signals, with a combination of effects (baseline wandering, amplitude modulation, frequency modulation, etc.), but with high SNR (around 100:1), the best performing algorithms were again, as in the case of the perfect sinusoids, the forward-backward AR approach with no windowing, the least-square AR with no windowing, the eigenvector-based method and the root MUSIC algorithm. However, for SNR in the order of 10:1 and lower, the AR estimators deteriorate and are no longer

among the best-performing ones, and then Hilbert, TF moment and root MUSIC ranked the best.

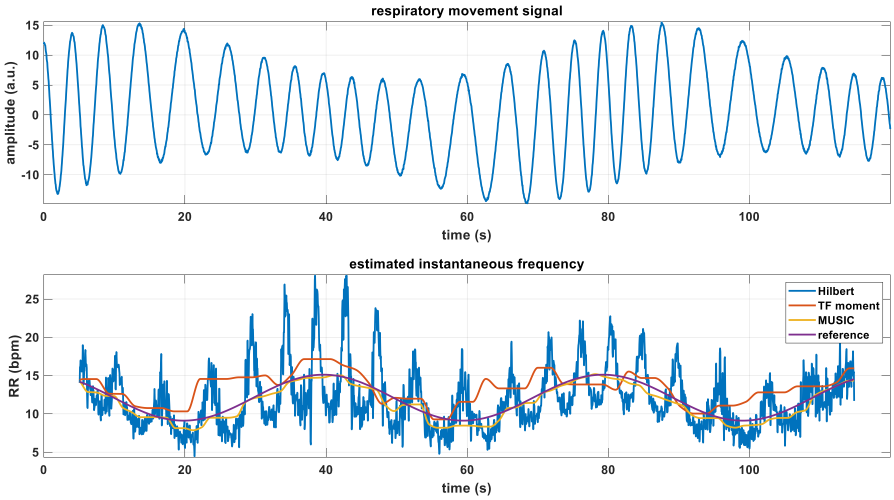

Figure 3. Synthetic realistic respiration movement signal (top panel) and the instantaneous frequency signals estimated by the best-performing algorithms, along with the known reference signal.[3]

Figure 3 illustrates an example (one run) of respiration movement signal generated with effects resembling the real-world signal (upper plot) and the resulting instantaneous frequency signals estimated by the best performing methods: Hilbert, TF moment and root MUSIC. Observe that root MUSIC (yellow) estimated a respiration rate signal looking alike the ideal reference (magenta) in the lower plot of Figure 3.

Figure 4 shows that, even when the root MUSIC algorithm implementation has an execution time longer than TF moment and Hilbert, this is not that critical, taking less than 200 ms to execute a run. That is, the estimation of the 2-min instantaneous respiration frequency takes less than 200 ms, although the TF moment and the Hilbert implementations execute faster. The maximum absolute error, which is the maximum error in the whole length estimated signal, of the root MUSIC algorithm, however, is much lower than the other two (i.e. TF moment and Hilbert). That is, less than 2 bpm as an average, for root MUSIC, versus averages of 5 bpm and more than 14 bpm, for TF moment and Hilbert, respectively. The mean-squared error (MSE) of the root MUSIC

[3] Created by the authors.

algorithm (less than 0.5 bpm^2 as an average) is also much lower than the MSE of the TF moment algorithm (4 bpm^2 as an average) and the Hilbert algorithm (~ 10.5 bpm^2).

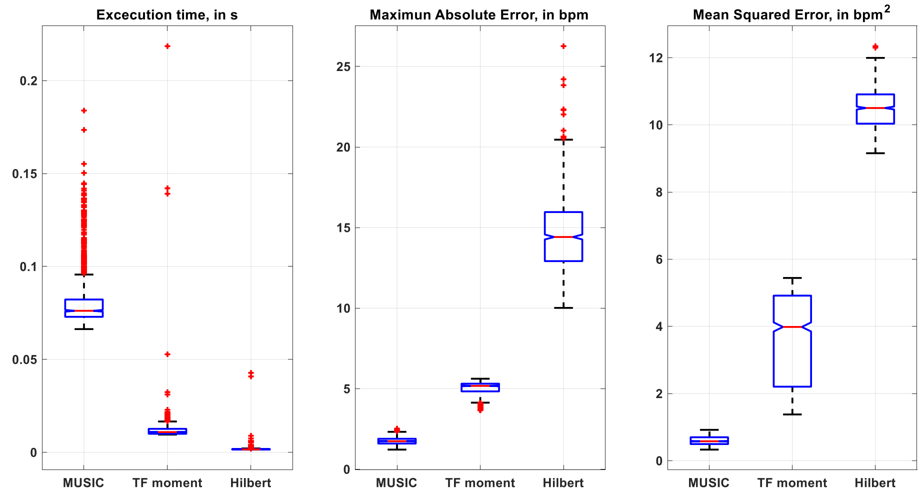

Figure 4. Box-plots comparing the best-performing algorithms according to the execution time, the maximum absolute error and the mean-squared error after 1000-run simulation.[4]

The box plots obtained for 1 000 runs (Figure 4) suggest that the differences between the root MUSIC algorithm and the others under assessment here, according to MAE and MSE, are statistically significant. This was corroborated by different statistical tests obtaining all *p*-values of 0 or very close to 0. Therefore, after this experimentation, we can recommend the root MUSIC algorithm for estimating the instantaneous respiration frequency from respiratory volume surrogated signals.

3.3. Instantaneous frequency estimators in the presence of real-world signals

We, finally, estimated the respiration rate signals from real-world respiratory movement signals (by using a thermistor close to the mouth and the nose) provided for the CLAIB 2019 Scientific Challenge to be held in Cancun, Mexico, in October, 2019. This dataset comprises six respiratory movement (reference) signals and six related instantaneous frequency (inst_freq) signals for two subjects (s11 and s34) and three conditions (resting and moving, with a metronome guiding a 0.2-Hz respiration rate, and spontaneous breathing at rest). Then, we computed the mean-squared error of the estimated

[4] Created by the authors.

instantaneous frequency signals from the reference signals, assuming the inst_freq signals as references. Table 1 shows the results.

Table 1. Mean-Squared Error of the implemented estimators of instantaneous respiratory frequency, using the data provided for the CLAIB 2019 Scientific Challenge, Cancun, Mexico, Oct. 3-5, 2019.[5]

	Mean-Squared Error (in bpm²) of the instantaneous frequency estimators (comparing with the given inst_freq 'reference')								
Subjects and conditions	**Burg-AR**	**Forward-backward**	**GL-AR**	**LS-AR**	**Eingen vector**	**Root MUSIC**	**TF moment**	**Hilbert**	**Peaks**
S11-metro-02Hz-resting	**0.34**	**0.35**	**0.33**	**0.35**	0.43	**0.35**	1.97	7.58	0.56
S11-metro-02Hz-moving	**0.67**	0.72	**0.69**	0.71	0.87	**0.70**	2.97	23.75	337.46
S11-sponta-resting	1.84	1.21	1.94	**1.18**	**1.07**	1.23	2.69	39.16	**0.78**
S34-metro-02Hz-resting	0.66	**0.55**	0.66	**0.55**	**0.54**	**0.53**	1.93	34.31	7.45
S34-metro-02Hz-moving	**0.56**	**0.56**	**0.56**	**0.56**	0.57	**0.56**	3.20	18.17	2.22
S34-sponta-resting	**4.85**	5.39	**5.24**	5.31	5.29	5.32	6.73	143.45	**2.48**

We can see from Table 1 that there are several here implemented estimators of the instantaneous frequency signals reaching comparable results to the inst_freq provided with the dataset. Among the estimators, the auto-regressive based approaches (Burg, Geometric Latice, Least-Squares, and Forward-backward), the eigenvectors and the root MUSIC got the best performances, assuming the inst_freq signals provided as gold standards. Root MUSIC is always ranked among the best performing algorithms, reinforcing the idea of the previous results with the simulated signals (sections 3.1 - 3.2).

4. Conclusions

Different algorithms for instantaneous frequency estimation, or even the same algorithms with different input parameters, may obtain very different results.

Most algorithms have limited frequency resolution and are shift-variant, but the AR-like methods, based on the roots of the time-varying polynomials (e.g. AR, eigenvectors, root MUSIC), are very good in this regard. However, the AR estimators fail in the presence

[5] Created by the authors.

of certain level of noise (SNR<10) because it is almost impossible to obtain good instantaneous covariance estimations.

AR, eigenvectors, and root MUSIC algorithms are not as seriously affected by the values of the length of the analysis window (*T*), and the updating period (*dt*) as some other algorithms as the TF moment.

Some algorithms, as TF moment and Hilbert, exhibit an oscillatory behavior, generating peaks and oscillations not due to the nature of the signals from which the instantaneous frequency are estimated, but because of the estimation process itself.

The root MUSIC algorithm outperforms the others under assessment, showing its superiority for instantaneous respiratory frequency estimation from a volumetric surrogate signal. Therefore, root MUSIC is the algorithm of choice to estimate the time-varying respiration rate.

In future researches, we should implement and assess some other more recent high-resolution techniques, as the Iterative Sparse Asymptotic Minimum Variance Based Approach, and to extend their use beyond the mono-component signals. This study is needed for several applications, e.g. for estimating the time-varying instantaneous frequency of the alpha peak in the electroencephalography power spectrum.

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