INTERNATIONAL SYMPOSIUM CONSTRUCCIONS - STRUCTURES

Title

Criteria used for optimal sensor positioning problem solution during vibration measurements.

Title

Criterios empleados para la solución de la tarea de posicionamiento óptimo de sensores en mediciones de vibración.

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Abstract: Optimal Sensor Placement problem (OSP) in field vibration measurements is an optimal problem in which, an objective function related with dynamic characteristics of the structural system is minimized (or maximized). In this case, sensors positions are defined as discrete variable to be optimized, with a number of restrictions including the number of sensors and the coordinate range inside the structure, in which they could be placed. Several different criteria, or performance indexes leading to different objective functions have been used for solving the OSP problem, but with a common goal: obtain the maximum possible information about the dynamic behavior of structure. Among all strategies for solving OSP problem, highlight objective functions of the medium square error criterion (***MSE***), Modal assurance criterion (***MAC***), determinant of Fisher information matrix (***FIM***) and information entropy (***IE***). Also, objective functions based on on modal kinetic energy and modal deformation energy has been used. Paper describes and compare such criteria formulations w.r.t its results for a simple structure and extracts conclusions and recommendations.

Keywords: optimal sensor placement problem; structural health monitoring; MSE criterion, MAC criterion, FIM criterion, IE criterion.

Palabras Clave: problema de posicionamiento óptimo de sensores; monitoreo estructural; criterio MSE, criterio MAC, criterio FIM, Criterio IE.

1. Introduction

Currently, at least regular structural health monitoring, when not continuous, during exploitation period, depending on the importance of the pathway and bridge structure, is required for structural safety.

These regular structural monitoring procedures are based on assumes technologies usually depend on the adoption of increasingly reliable sensors suitable for the monitoring purposes. The **optimal sensor placement** plays a fundamental role in improvement the quality of health monitoring of civil engineering structures, process in which the sensor number is limited by its cost, while structures have multiples degree-of-freedom (DOF). However, the quality of the obtained information significantly depends on the numbers and positions of corresponding sensors (Allemang, 2003). Owing to the cost limitation, it is difficult and barely to place sensors in all appropriate positions. In this sense, deploying fewer sensors on the structures and acquiring more structure health information is a key issue. Especially, how to place sensors reasonably becomes one of the most importantly problems, which is known as optimal sensor placement (OSP).

Due to the above-mentioned reason, the OSP has received considerable attentions and has been investigated in different areas in the past decade. Several methods and *performance indexes* have been proposed by researchers for solving the OSP problem: the medium square error (***MSE***), modal assurance criterion (***MAC***), determinant of Fisher information matrix (***FIM***), information entropy (***IE***), modal kinetic energy (***MKE***) and modal deformation energy (***MDE***); which afterwards became itself objective functions or methods' basing. Section 3 and 4 summarizes them.

The OSP problem is usually a discrete optimization procedure. Because of that, meta-heuristic algorithms are well suited to face such problems, but here paper deals mainly with such performance indexes, which afterwards are constituted as basis of its objective functions (OF).

The structure of the paper is as follows. In section 2, the OSP problem formulation is described. In Section 3, several methods for solving OSP task are commented and complemented with several performance indexes described in section 4. Also, some numerical experiments are presented in a simple study case, developed to show the performance of each method/index on these two sections. Finally, conclusions are drawn in section 5. Acknowledgments and References are presented in Sections 6 and 7 respectively.

2. Optimal Sensor Placement problem formulation

The behavior of a structural system could be described as a multi-degree of freedom (*M-DOF*) structural system, using lineal dynamics and modal decomposition approach, which leads to a matrix form of the structural system motion equations as shown in eq. (1):

$$\boldsymbol{M} \cdot \ddot{z}(t) + \boldsymbol{C} \cdot \dot{z}(t) + \boldsymbol{K} \cdot z(t) = \boldsymbol{P}(t) \quad (1)$$

where **M, C** and **K** are the mass, damping and stiffness matrices respectively; $\ddot{z}(t), \dot{z}(t),$ and $z(t)$ are the corresponding system masses acceleration, velocity and displacement and $\boldsymbol{P}(t)$ represent the external force function applied to the system. For a structural system with **N** DOF, matrix orders for **M** modes should be $\boldsymbol{N\ x\ M}$.

For $\boldsymbol{P}(t) = 0$, and considering low damping values, as usual in structural systems, eq. (1) become the free vibration motion equations of the system:

$$\boldsymbol{M} \cdot \ddot{y}(t) + \boldsymbol{K} \cdot y(t) = \boldsymbol{0} \quad (2)$$

The generalized eigenvalue problem formulation of such a ***N**-DOF* linear system containing the ***M*** modes, with mass and stiffness matrices and mode shapes, could be written as follows:

$$\boldsymbol{K} \cdot \boldsymbol{\Phi} = \boldsymbol{M} \cdot \boldsymbol{\Phi} \cdot \boldsymbol{\Lambda} \quad (3)$$

where $\boldsymbol{\Phi} \in \Re^{\boldsymbol{NxM}}$ is the mass-normalized mode shape matrix; $\boldsymbol{\Lambda} \in \Re^{\boldsymbol{NxM}}$ is the diagonal eigenvalue matrix consisting of the eigenvalues λ_i (*i = 1, 2, 3, …* ***M***). $\boldsymbol{\Phi}$ is composed of column vectors representing mode shapes vectors, and the elements of column vector correspond to possible locations of single axes acceleration sensors.

The goal of OSP is to select k rows from the mode shape matrix $\mathbf{\Phi}$, in such a way, that objective function assumed for solving the problem, could be valued as optimal as possible (Yin, et al., 2017). That is, for $\mathbf{\Phi} \in \mathfrak{R}^{NxM}$, find a permutation matrix $\mathbf{P} \in \mathfrak{R}^{NxN}$ so that (Yin, et al., 2017):

$$\boldsymbol{P} \cdot \mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_S \\ \widehat{\mathbf{\Phi}}_S \end{bmatrix}, \mathbf{\Phi}_S \in \mathfrak{R}^{kxM} \tag{4}$$

where $\mathbf{\Phi}_S$ is the modal sub-matrix which is measured by the selected single axe acceleration sensors; k is the number of sensors.

As mentioned above, each row of modal matrix $\mathbf{\Phi}$ represents a DOF, that is a location which can be placed a single axis acceleration sensor. Usually, only few from all global mode shapes are enough and selected for identification, and for OSP problem solution.

Salamanca (Salamanca Figueroa, 2018) also formulate the OSP problem referencing Sun & Büyüköztürk (Sun & Büyüköztürk , 2015) as an optimal problem in which an objective function related with dynamic characteristics of the structural system is minimized. In this case, sensors positions are defined as discrete variable to be optimized, with a number of restrictions including the number of sensors and the coordinate range inside the structure, in which they could be placed. Problem OSP is defined according to this approach as:

$$min\ f(v) \tag{5a}$$

$$g(v) = n \tag{5b}$$

$$v^{lb} \leq v \leq v^{ub} \tag{5c}$$

$$v \in Z^{+} \tag{5d}$$

where v is a vector of sensor positions defined by integer numbers $v = [\theta_1, \theta_2, \ldots, \theta_n]$, $f(v)$ is the objective function, $g(v)$ is the total number of positions during the optimization process, n is the number of sensors, v^{lb} y v^{ub} represent the lower and upper bounds in sensors positions, respectively, and Z^{+} is the set of positive integer numbers.

3. Methods for Optimal Sensor Placement problem solution during vibration measurements

Although since the very beginning of the studies for solving the OSP problem (70's), multiples methods have been proposed by researchers, most of them have a common goal: identify on the more exact way, the dynamic behavior of the under-studying structure (Salamanca Figueroa, 2018).

Depending on the optimization technique used, methods for solution the OSP problem could be grouped on three categories: *direct classification methods, iterative elimination and expansion*, and *combinatory optimization algorithms* or *heuristic methods* (Tong, et al., 2014).

Direct Classification Methods

This group' methods are based on assigning a performance index to each candidate-for-optimal-sensor position. Locations are classified according to the value of assigned index, and positions with the higher index are selected to integrate the optimal sensor setup. Simplicity of this optimization procedure, makes techniques under this groups the faster way to select optimal sensor positions.

The most important drawback of these procedures is the trend to locate the optimal sensors positions in a reduced range of the structure DOF's, due to the fact that the OF used takes the highest values in the optimal excitation points, therefore the resulting sensor position distributions coming from these methods are poor and do not ensure a precise mode shape identification after measurements.

Two of the exponents of this group' method are the *method of the eigenvector product* (EVP) and the *driving point residual method* (DPR).

EVP methods selects as optimum positions those which maximize vibration energy from sensors signal. Vector EVP is computed from (Doebling, 1996):

$$EVP_i = \prod_{j=1}^{M} |\phi_{ij}| \quad (6)$$

being M: the number of modes, ϕ_{ij}: i^{th} element from the j^{th} mode in mode shape matrix in the FE model of the structure, and EVP_i is the objective function value for i^{th} sensor position.

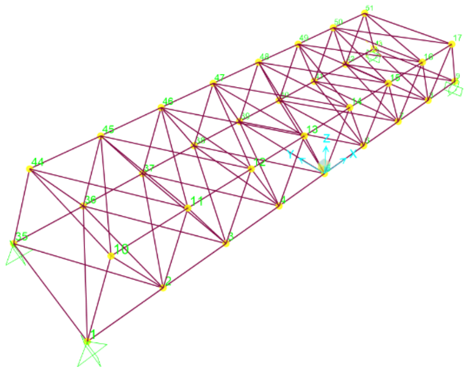


Fig. 1 Truss type bridge model (18 m span, 6 m wide), in CSi - SAP 2000

In turn, DPR method raises the following formulation:

$$DPR_i = \prod_{j=1}^{M} \frac{\phi_{ij}^2}{\omega_j} \quad (7)$$

In which, practically the same input data are used, with ω_j: angular frequency for the jth mode shape, and DPR_i is the objective function value for i^{th} sensor position.

Results of the application of **EVP** and **DPR** methods to a Truss type Bridge, 18 m span, and 6 m wide (see Fig. 1), are shown on Fig. 2 and Fig. 3 respectively, for the first 4 mode shapes.

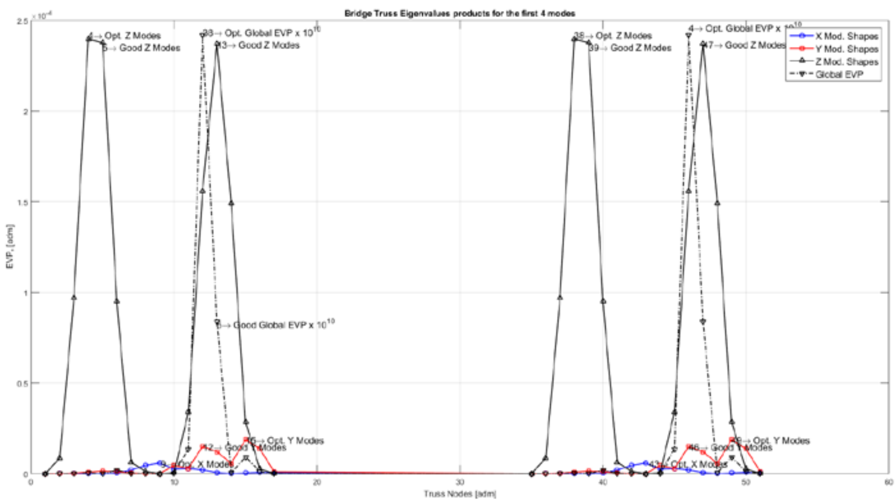


Fig. 2 Node labels for optimal sensor locations according to EVP Method (source: own elaboration).

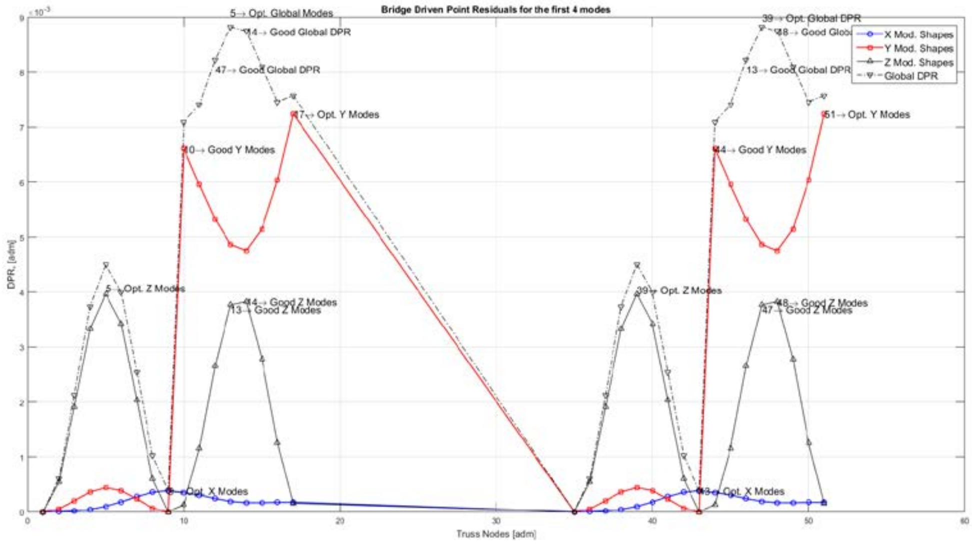
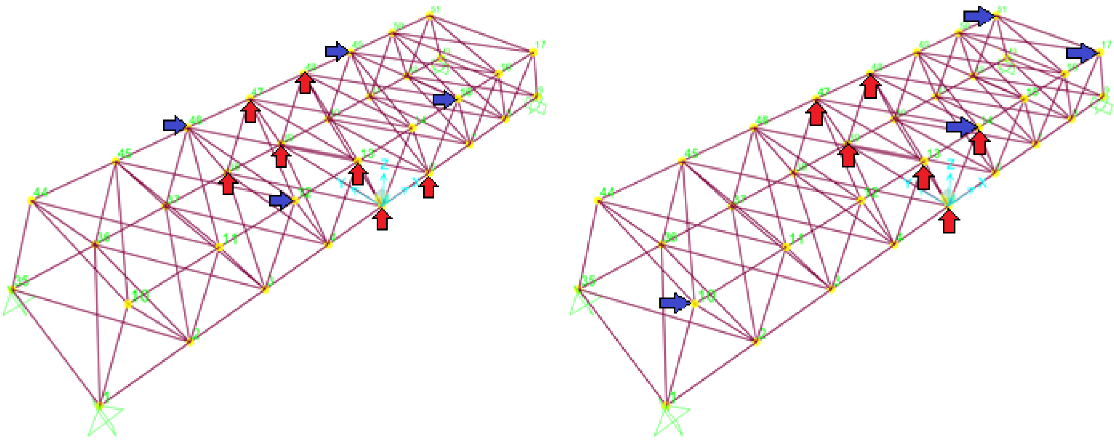


Fig. 3 Node labels for optimal sensor locations according to DPR Method (source: own elaboration).

According to **EVP** methods, best position of sensors could be seen at Fig. 4a while according to **DPR** methods, best position of sensors could be seen at Fig. 4b, highlighted in red for global vertical flexural mode shapes coordinates and in blue, for global flexural horizontal mode shapes coordinates.



a *b*

Fig. 4 Nodes for optimal sensor locations according to: a. EVP Methods, b. DPR Methods. Highlighted in red for global vertical flexural mode shapes, highlighted in blue, for global flexural horizontal mode shapes (source: own elaboration).

When comparing both methods, best nodes for global vertical mode shapes seems to have matching positions in the vicinity of half bridge span, while for global horizontal mode shapes, resulting best node positions do not match for both methods. If a global EVP or DPR are performed, then results trend to have a better approximation.

Methods of iterative elimination and expansion

Iterative elimination methods start fixing a determined number of candidate-to-optimum locations, and assigning to each one, an index obtained from the objective function evaluation of the method itself, which evaluate the global contribution of each position to structure response. Position with the lower index is discarded, and the process is again repeated until the desired number of positions is obtained. Such positions conform the optimal sensor configuration. In this group are mentioned the kinetic energy method (**KEM**) and effective independency method (**EfIM**). Despite these methods operate in different ways, performance of **KEM** and **EfIM** in **OPS** problem solutions are similar by effectiveness and precision (Salamanca Figueroa, 2018).

Main drawback of **EfIM**, which is common to all procedures based on **FIM**, is that the number of sensors should, at least, equals the number of modes to be identified, to avoiding singularities of autovector matrix. Conversely, iterative expansion methods broaden the number of sensor positions until the number of desired optimal positions is reached.

KEM demands mass matrix as input or its upper (lower) triangular decomposition and usually return a sensor configuration in only one side in symmetric structures, with worse performance.

Heuristics Methods

As mentioned above, **OSP** problem is usually a discrete optimization procedure, in which meta-heuristic algorithms are well suited to face such problems. Although these algorithms demand a higher computational capacity and cost, they offer global optimum and precise solutions, which exceed the drawbacks of iterative elimination and expansion methods.

Combinatory problem of sensor placement is based in the search of the best possible combination out of ***r*** sensors at ***n*** possible-candidate positions, being $n!\ /\ (r!\ (n{-}r)!)$ the number of possible combinations. Hence the approach is the identification of the optimal sensor configuration by aleatory search. Nevertheless, this approach has been exceeded for a best efficiency, due to the fact that initially processed information on each aleatory configuration is not used for finding best sensor configurations in future search.

4. Performance Indexes and objective functions

Several different criteria have been used as objective functions for solving the OSP problem, but with a common goal: obtain the maximum possible information about the dynamic behavior of structure. Among all strategies for solving OSP problem, highlight objective functions of the medium square error (***MSE***), Modal assurance criterion (***MAC***), determinant of Fisher information matrix (***FIM***) and information entropy (***IE***). Also, objective functions based on modal kinetic energy (***MKE***) and (***MDE***) modal deformation energy (Salamanca Figueroa, 2018) has been used, but these criteria seem to be similar to ***FIM***, and demand more input parameters as mass and stiffness matrices.

MSE determines the square medium error between mode shapes extracted from a finite element (FE) model and correspondent from sensor measurements at selected locations. Usually, each mode is normalized with respect to standard deviation σ of the sensor' obtained response. ***MSE*** could be expressed as follows:

$$MSE = \sum_{i=1}^{N} \frac{\frac{1}{\sigma^2}\sum_{j=1}^{n}\left(\phi_{ij}^{SP} - \phi_{ij}^{FE}\right)}{n} \tag{8}$$

where *i* represent mode shapes, *j* is the component of each vector associated to a mode shape, ϕ_{ij}^{SP} are mode shapes identified by sensor positions, ϕ_{ij}^{FE} are mode shapes extracted from a FE model, n is the number of components of mode shape vectors (i. e. number of sensor positions), *N* is the number of combinations of sensor pairs and σ^2 is the output variance.

FIM is a mathematic tool which historically has been adapted to multiple applications in statistic scope, which has been used to quantify the amount of information relative to an unknown parameter, content in a modeled distribution by an aleatory observable variable. Basic definitions and concepts related with FIM could be found on (John D., n.d.).

OSP problem can be solved by maximizing ***FIM*,** correlating the information extracted from measurements from sensors placed in the structure with information of the FE model (Salamanca Figueroa, 2018). Mode shapes extracted should be linearly independents and spatially differentiable, which implies that for any time instant, sensor response equation be:

$$u_S = \phi_S q \tag{9}$$

where q is the vector of modal coordinates, ϕ_S is the mode shape matrix from **FE** model, at sensor positions.

Introducing a response modification:

$$u_S = H(q) + N = \phi_S q + N \tag{10}$$

where H is sensor measurements and N, a vector representing the variance of stationary Gaussian white noise ψ_0^2, aleatory signal, which values do not have any relation in time, and with a density function is a normal Gauss distribution.

Covariance matrix for an impartial efficient estimator could be written as:

$$P = E[(q - \hat{q})(q - \hat{q})^T] = [\left(\frac{\partial H}{\partial q}\right)^T [\psi_0^2](\frac{\partial H}{\partial q})]^{-1} \tag{11}$$

where E is the expectation. Although this formulation assumes that displacements are measured, similar formulation and results would be obtained for velocity or accelerations measurements. With $H(q) = \phi_S q$, covariance matrix is:

$$P = E[\phi_S^T (\psi_0^2)^{-1} \phi_S]^{-1} = Q^{-1} \tag{12}$$

being $\boldsymbol{q}$ the Fisher information matrix (FIM). Maximizing $\boldsymbol{Q}$ is equivalent to minimizing covariance matrix, therefore, develop a better estimation of $\hat{\boldsymbol{q}}$. FIM also could be expressed as:

$$Q = \frac{1}{\psi_0^2} \phi_S^T \phi_S = \frac{1}{\psi_0^2} A_0 \tag{13}$$

Therefore, minimizing P should maximize a norm of A_0, and due to the fact that the matrix determinant does not depend on sensor noise, usually researches refer to FIM as $A_0 = \phi_S^T \phi_S$. Also, in terms of contribution to each DOF, A_0 could written as:

$$A_0 = \sum_{i=1}^{S} \phi_S^{iT} \phi_S^i = \sum_{i=1}^{S} A_i \tag{14}$$

where $\boldsymbol{\phi_S^i}$ is the i^{th} row of ϕ_S, $\boldsymbol{i}$ is the i^{th} DOF or the i^{th} sensor position.

Although the number of sensors should be at least equal to the number of mode shapes to be identified, which is the major drawback of FIM procedure, in practice the number of sensors is usually greater than the possible minimum, for a proper identification of mode shapes.

So, the objective function of FIM, applying continuous health monitoring could be w ritten as (Abt & Welch, 1988):

$$f = max.\det(A_0) = max.\det(\phi_S^T \phi_S) \quad (15)$$

Results of the application of FIM index to sensor positions obtained from application of EVP and DPR methods to the Truss type Bridge presented above, are summarized in the following Table no. 1.

Table no. 1 Results in application of FIM index to two sensors setups.

Method for sensor placement	Nodes selected for Vertical mode shape coordinates	Determinant of FIM value	Nodes selected for Horizontal mode shape coordinates	Determinant of FIM value
EVP	[4 5 13 38 39 47]	1.2143	[12 15 46 49]	0.7304
DPR	[5 13 14 39 47 48]	1.2676	[10 14 17 51]	1.2155

For vertical mode shapes coordinates, both sensors' placements methods seem to perform similarly, with a little better result for sensors' positions determined by DPR method. For horizontal mode shapes coordinates, sensors positions determined by DPR method perform much better than EVP method.

Information entropy (IE) or Shannon entropy, is well known as the unique measurement of probabilistic uncertainty of model parameters. It depends on FIM determinant and could be used in the OSP and also for estimating structural dynamic parameters during system identification in non-linear models (Papadimitriou, et al., 2000).

As ***IE*** is a measurement of uncertainty of estimated parameters, is selected such sensor configuration that minimize ***IE***. This criterion could be expressed as:

$$f_{optimum} = min.H(L;\ \theta_0, \Sigma) = min.[\frac{1}{2}N_0 \ln(2\pi) - \frac{1}{2}\ln [det\boldsymbol{Q}(\boldsymbol{L};\theta_0,\Sigma)]] \quad (16)$$

where θ is the vector of free parameters, that should be estimated by measured data, N_0 is the number of observed DOF, $\boldsymbol{L}$ is the observation matrix, Σ is the covariance of the aleatory gaussian vector used to model the prediction error $\boldsymbol{e}_k(\boldsymbol{\theta})$.

It's been demonstrated that results of IE aproach in OSP problem solutions are similar to those obtained by FIM aproach (Salamanca Figueroa, 2018). Hence, also the number of sensor, should be at least equal to the number of mode shapes to be identified, as ***IE*** aproach uses ***FIM***.

The modal assurance criterion (***MAC***) is defined as a scalar constant relating the degree of consistency (linearity) between one modal and another reference modal vector as follows (Allemang, 2003):

$$MAC_{cdr} = \frac{\left|\sum_{q=1}^{N_0} \Psi_{cqr}\Psi_{dqr}^*\right|^2}{\sum_{q=1}^{N_0} \Psi_{cqr}\Psi_{cqr}^* \sum_{q=1}^{N_0} \Psi_{dqr}\Psi_{dqr}^*}, \quad (17a)$$

or

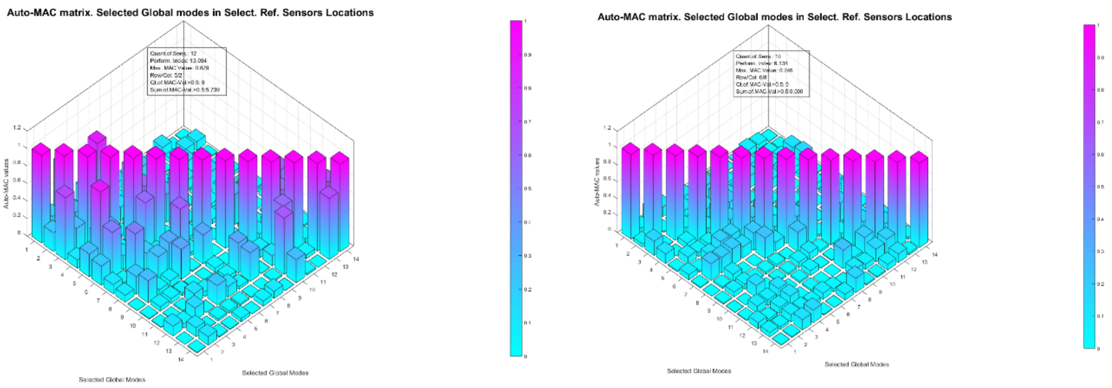
$$MAC_{cdr} = \frac{\left|\{\Psi_{cr}\}^T\{\Psi_{dr}^*\}\right|^2}{\{\Psi_{cr}\}^T\{\Psi_{cr}^*\}\{\Psi_{dr}\}^T\{\Psi_{dr}^*\}} \quad (17b)$$

where N_0 correspond to the number of measurement locations on the structure. Eq. 17 implies that the *modal vector* ***d*** is the reference to which the *modal vector* ***c*** is compared.

The *modal assurance criterion* takes on values from zero – representing no consistent correspondence, to one – representing a consistent correspondence. In this manner, if the modal vectors under consideration truly exhibit a consistent, linear relationship, the modal assurance criterion should approach unity. Fig. 4 a, b shows two examples of MAC matrix values representation using 3D plots. Fig. 4a shows a non-optimal sensor configuration for measurements, while Fig. 4 b shows MAC matrix values obtained after solving OSP problem. It's clearly seen from left plot, 9 off-diagonal values above 0.5, meaning that 9 pair of selected mode shapes would be non-distinguishable from that sensor configuration for measurements, while in the right plot there are no MAC matrix values in off-diagonal positions above 0.5, which would result in a clear identification of selected mode shapes from measurement. Last situation is the typical MAC matrix plot, after solving OSP problem.

This means that optimal sensor configuration would make MAC matrix with ones on its diagonal and with close-to-zero numbers in off-diagonal elements, therefore an indicator of how optimum is a sensor placement configuration would be the value of the off-diagonal elements of MAC matrix.

Based on off-diagonal element values, two objective functions for OSP were proposed (Carne & Dohmann, 1995): first determine which is the largest off-diagonal MAC matrix element. So, for a given sensor configuration, denoted by v:



a *b*

Fig. 4 MAC matrix values representation using 3D plots: a) MAC matrix values from a non-optimal sensor configuration placement for measurements, b) MAC matrix values obtained after solving OSP problem (source: own elaboration).

$$f_1(v) = max_{i \neq j}\{MAC(v)_{ij}\}_{i,j=1,2,\ldots,p} \tag{18}$$

where $\boldsymbol{p}$ is the selected number of global mode shapes to be identified by the measurement process with the OSP. Second objective function proposed is the sum of the square of the off-diagonal MAC matrix values:

$$f_2(v) = \sum_{i=1, j=1\ (i \neq j)}^{p} \left[MAC(v)_{i,j}\right]^2 \tag{19}$$

In OSP problem, these functions should be minimized, and for finding the optimal sensor placement configuration, both should be minimized:

$$f(v) = min.\, f_1(v) \tag{20}$$

$$f(v) = min.\, f_2(v) \tag{21}$$

Results of the application of MAC performance index to sensor positions obtained from application of EVP and DPR methods to the Truss type Bridge presented above, are summarized in the following Fig. no. 5.

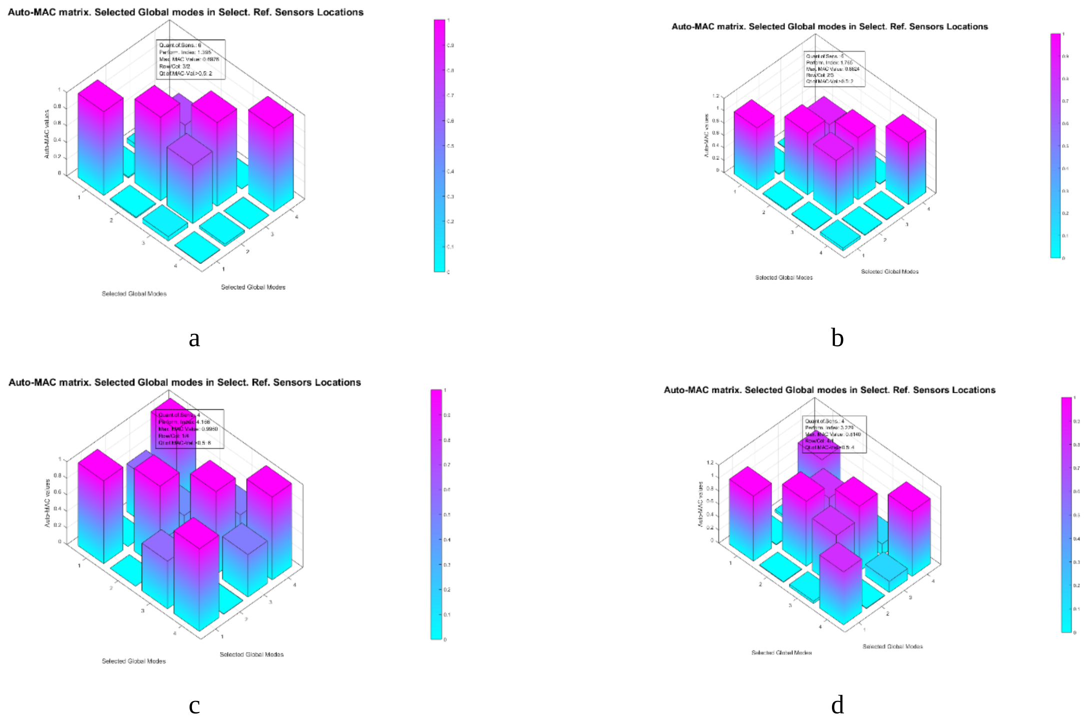


Fig. 5 Results in application of FIM index to two sensors setups. Nodes selected for: a. Vertical mode shape coordinates using EVP method, b. Idem. using DPR method, c. Horizontal mode shape coordinates using EVP method, d. Idem using DPR method.

Again, results seem to perform contradictory: out of four sensor setups, none perform satisfactorily. The best sensor configuration is for vertical sensors which placement was selected by EVP method, and for such configuration, two of off-diagonal MAC values are above 0.5, Therefore, it can be inferred that this criterion is more restrictive than those discussed above.

From the results of the application of the MAC criterion it is evident that modes 2 and 3 are not clearly identifiable if only sensor positions in the vertical direction are chosen according to the EVP or DPR criteria (see Fig. 5a, b). The horizontal sensor configurations derived from the application of both criteria perform worse in clearly identifying modes 2 and 3 as well as 1 and 4 (See Fig. 5c, d).

5. Conclusions

Criteria for solving the OSP problem in SHM are summarized in this paper. The general purpose of this study is to compare the performance of the different criteria/indexes used to determine the optimal arrangement of sensors in a structure, which provides the maximum dynamic information of this structure, given a limited number of sensors. The OSP problem as it is known, is first formulated as a constrained discrete (integer) optimization problem, where the integer variables denote the possible locations of the sensors.

Several criteria used are investigated and the results of their application in the structure of a bridge in the form of space truss are discussed. The numerical results show that the first criteria exposed (EVP and DPR) can serve to offer an initial idea of the optimal positioning, or serve as a complement to the rest. The FIM maximization criteria and the combined variants of the MAC-based indices tend to perform more demanding and robust in determining the optimal sensor positions.

It is observed in the numerical results that the OSP configuration depends on the formulation of the objective function used in the optimization process. The objective functions based on the sum of the least squares of the values of the off-diagonals MAC matrix elements has a faster and more robust convergence compared to the case in which the maximum value of the off-diagonals of the MAC matrix elements is used.

Future studies will focus on SHM applications in the field and damage detection based on the sensor arrangement obtained by the proposed OSP algorithm. Other practical problems of this task have also been left out, such as:

1. The decision between how many reference sensors vs. Place mobile sensors, for a limited total number of sensors, when considering the multi-setup technique during SHM.
2. Possibility of evaluating a discretization of the domain of the positions to be evaluated during the solution of the task of the optimal positioning of the sensors by means of discrete or metaheuristic optimization, among others.

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